

H-infinity with pole placement constraint in LMI region for a buck-converter driven DC motor

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Abstract— This paper presents a H-infinity synthesis with pole clustering based on LMI region schemes to control the speed of a DC motor. The dynamic system composed of converter/motor is considered in this investigation and derived in the state-space and transfer function forms. The performance of the control schemes is examined in terms of time response specifications of angular velocity, duty cycle input energy and armature current. The implementation results show that the regional pole placement known as LMI region combined with design objective in H-infinity controller guarantee a fast angular velocity tracking capability with very minimal controlled duty cycle.

Keywords—component; H-infinity, LMI region, buck-converter, DC motor speed.

I. INTRODUCTION

Investigations to improve the switching strategy in PWM signals to control the speed of DC motor have received extensive attention recently. The conventional cases rely on hard switching strategy which causes unsatisfactory dynamic behavior and the resulting trajectories exhibit a very noisy shape. This also causes large forces acting on the motor mechanics and also large currents which detrimentally stress the electronic components of the motor as well as of the power supply [1]. Since it is usually necessary to add a power supply component, anyway, this contribution shall present a control for the entire system of buck-converter/DC motor. The combination of DC to DC power converters with DC motors has been reported in [2]. In particular, the composition of a buck converter with a DC motor with a flatness based approach has been proposed in [3]. They show that smooth DC output voltages and currents with very small current ripple can be generated due to the fact that the converter contains two energy storing elements, a coil and a capacitor. In addition, the optimal and intelligent controls also have been introduced to generate a smooth trajectory of DC motor speed [4,5]. In this respect, an important issue is the circuit design of the converter in order to obtain, at any time, a high power conversion rate when tracking smooth reference trajectories of the angular velocity. Therefore, this stage is discussed in detail.

In this study, H-infinity synthesis with pole clustering based on LMI techniques is used to control the angular velocity of buck-converter driven DC motor with very minimal duty cycle energy. In order to design the controller, the linear model of buck-converter driven DC motor system is obtained based on the circuit shown in Figure 1. The reason for choosing H-

infinity synthesis is because of its good performance in handling with various types of control objectives such as disturbance cancellation, robust stabilization of uncertain systems, input tracking capability or shaping of the open-loop response. Nevertheless, the weakness of H-infinity controller is in handling with transient response behavior and closed-loop pole location instead of frequency aspects [6]. As we all know, a good time response specifications and closed-loop damping of buck-converter driven DC motor system can be achieved by forcing the closed-loop poles to the left-half plane. Moreover, many literatures have proved that H-infinity synthesis can be formulated as a convex optimization problem involving linear matrix inequalities (LMI) [7]-[9]. In this case, the normal Riccati equation with inequality condition was used. This behavior will give wide range of flexibility in combining several constraints on the closed loop system. This flexible nature of LMI schemes can be used to handle H-infinity controller with pole placement constraints. In this work, the pole placement constraints will refer directly to regional pole placement [10]. It is slightly difference with point-wise pole placement, where poles are assigned to specific locations in the complex plane based on specific desired time response specifications. In this case, the closed-loop poles of buck-converter driven DC motor system are confined in a suitable region of the complex plane. This region consists of wide variety of useful clustering area such as half-planes, disks, sectors, vertical/horizontal strips, and any intersection thereof [10]. Using LMI approach, the regional pole placement known as LMI region combined with design objective in H-infinity controller should guarantee a fast input tracking capability with very minimal duty cycle energy.

The rest of this study is structured in the following manner. The next section provides a description of the linear model of buck-converter driven DC motor system in a state-space form. In section 3, the design of H-infinity controller with pole placement constraint is explained. Simulation work is reported in Section 4. Finally, concluding remarks are offered in the last section.

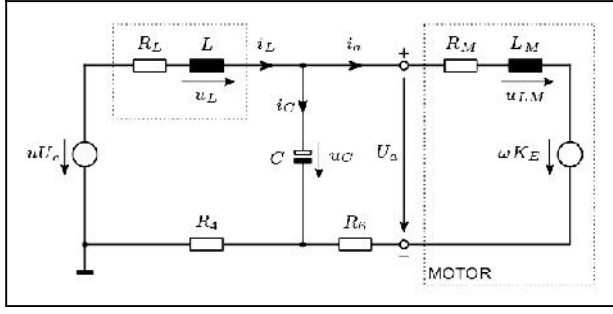


Figure 1. Layout of Buck-Converter with DC Motor System.

II. MODELLING OF BUCK-CONVERTER WITH DC MOTOR

A simplified model of the overall system buck converter/DC motor is shown in Figure 1 [1]. The switching devices have been replaced by an ideally switched voltage source. This is indicated by the multiplication of U_e with the switching variable $u \in \{0,1\}$. An additional resistance R_L has been added to the model in order to take into account the ohmic resistance of the coil windings. The motor has been modeled by an inductance L_M with ohmic resistance R_M and electromagnetic voltage source ωK_E , where the parameters have been determined as $L_M = 8.9$ mH, $R_M = 6$ Ω , and $K_E = 0.0517$ V/rad/s. For the converter, a switching frequency of $f = 45$ kHz has been used. As input voltage $U_e = 24$ V has been used since this is the maximum voltage of the DC motor. In this study, the buck converter circuit with coil inductance, $L = 1.33$ mH, coil resistance, $R_L = 0.2$ Ω and capacitance, $C = 470$ μ F is considered. State space modeling complete with mechanical equation that describes the dynamics of the motor shaft, a linear fourth order system is obtained as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \quad (1)$$

where

$$\mathbf{x} = [i_L \ u_C \ i_a \ \omega]^T \quad (2)$$

$$\mathbf{A} = \begin{bmatrix} -\frac{R_L}{L} & -\frac{1}{L} & 0 & 0 \\ \frac{1}{C} & 0 & -\frac{1}{C} & 0 \\ 0 & \frac{1}{L_M} & -\frac{R_M}{L_M} & -\frac{K_E}{L_M} \\ 0 & 0 & \frac{K_M}{J} & 0 \end{bmatrix} \quad (3)$$

$$\mathbf{B} = \begin{bmatrix} \frac{U_e}{L} & 0 & 0 & 0 \end{bmatrix}^T \quad (4)$$

The moment of inertia of the motor with tacho generator has been determined to be $J = 7.95 \times 10^{-6}$ kg m² and $K_M = 0.0517$

Nm/A. In (2) to (4), the very low measurement amplifier resistances R_4 and R_6 have been neglected. According to [11], the discrete input $u \in \{0,1\}$ can be replaced with the duty ratio $\delta \in \{0,1\}$ when using a PWM-strategy to generate δ from an analog input signal. Hence, for the given setup we may refer to a so-called averaged dynamic model, given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\delta \quad (5)$$

where the new input is the duty cycle δ . The output is given by

$$\omega = [0 \ 0 \ 0 \ 1]\mathbf{x}$$

Note that this linear system description is valid only as long as it can be ensured that no saturation effects occur in the coil. Otherwise the inductance L would depend nonlinearly on the current i_L .

III. DESIGN OF H-INFINITY CONTROLLER WITH LMI REGION

In this study, an integral state feedback control is used as a platform to design the proposed controller. The block diagram of integral state feedback control is shown in Figure 2.

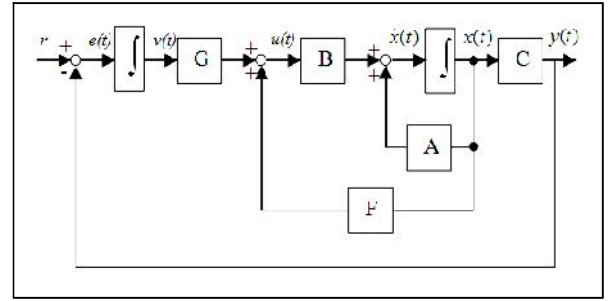


Figure 2. Block diagram of integral state feedback control.

The main objective of the proposed controller is to find the gain parameter matrix, F and G such that it fulfills the design requirement. From the block diagram of Figure 2, the control input of the system is derived as follow

$$u(t) = Fx(t) + Gv(t) \quad (6)$$

where $v(t) = \int e(\tau) d\tau$ and $e(t) = r - y(t)$

Using new state variable $x_e = [x^T \ v]^T$ and equation (6) the representation of state space equation can be rewrite as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \quad (7)$$

$$e(t) = r - Cx(t)$$

Next, at the steady state condition as $t \rightarrow \infty$, the state space equation can be written in the following form

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(\infty) \\ v(\infty) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(\infty) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \quad (8)$$

$$0 = r - Cx(\infty)$$

By subtracting (7) to (8), the state space form is converted to

$$\begin{aligned} \dot{\tilde{x}}_e(t) &= \tilde{A}\tilde{x}_e(t) + \tilde{B}_2\tilde{u}(t) \\ \tilde{e}(t) &= \tilde{C}_1\tilde{x}_e(t) \end{aligned} \quad (9)$$

where

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad \tilde{B}_2 = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \tilde{x}_e = \begin{bmatrix} \tilde{x} \\ \tilde{v} \end{bmatrix} = \begin{bmatrix} x - x(\infty) \\ v - v(\infty) \end{bmatrix} \\ \tilde{C}_1 &= [-C \quad 0], \quad \tilde{e}(t) = e - e(\infty) \end{aligned}$$

Then, the new control input function is described as follow

$$\tilde{u}(t) = F\tilde{x}(t) + G\tilde{v}(t) = K\tilde{x}_e(t) \quad (10)$$

Finally, a closed loop state space equation with controller gain, K can be obtained below

$$\begin{aligned} \dot{\tilde{x}}_e(t) &= \tilde{A}_{cl}\tilde{x}_e(t) + \tilde{B}_1w \\ \tilde{y}(t) &= \tilde{C}_1\tilde{x}_e(t) + \tilde{D}_{11}w + \tilde{D}_{12}u \end{aligned} \quad (11)$$

where

$$\tilde{A}_{cl} = (\tilde{A} + \tilde{B}_2K), \quad \tilde{B}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 \end{bmatrix}, \quad \tilde{D}_{11} = 1, \quad \tilde{D}_{12} = 0$$

and w is exogenous input disturbance or reference input to the system. Let $G_{yw}(s)$ denote the closed loop transfer function from w to y under state feedback control $u = Kx$. Then, for a prescribed closed loop H-infinity performance $\gamma > 0$, our constrained H_∞ problem consists of finding a state feedback gain K that fulfil the following objectives:

- (1) The closed loop poles are required to lie in some LMI stability region \mathcal{D} contained in the left-half plane
- (2) Guarantees the H_∞ performance $\|G_{yw}\|_\infty < \gamma$

Quote from the definition in [6], a subset \mathcal{D} of the complex plane is called an LMI region if there exist a symmetric matrix $\alpha \in \mathbf{R}^{m \times m}$ and a matrix $\beta \in \mathbf{R}^{m \times m}$ such that

$$\mathcal{D} = \{z \in \mathbf{C} : f_{\mathcal{D}}(z) < 0\} \quad (12)$$

where

$$f_{\mathcal{D}}(z) := \alpha + z\beta + \bar{z}\beta^T$$

Then, pole location in a given LMI region can be characterized in terms of the $m \times m$ block matrix

$$M_{\mathcal{D}}(\tilde{A}_{cl}, X_{\mathcal{D}}) := \alpha \otimes X_{\mathcal{D}} + \beta \otimes (\tilde{A}_{cl}X_{\mathcal{D}}) + \beta^T \otimes (\tilde{A}_{cl}X_{\mathcal{D}})^T \quad (13)$$

Quote from the theorem in [6], the matrix \tilde{A}_{cl} is \mathcal{D} -stable if and only if there exists a symmetric matrix X such that

$$M_{\mathcal{D}}(\tilde{A}_{cl}, X_{\mathcal{D}}) < 0, \quad X_{\mathcal{D}} > 0 \quad (14)$$

In this study, the region $S(\lambda, r, \theta)$ of complex numbers $x + jy$ such that

$$x < -\lambda < 0, \quad |x + jy| < r, \quad \tan \theta x < -|y| \quad (15)$$

as shown in Figure 3 is considered.

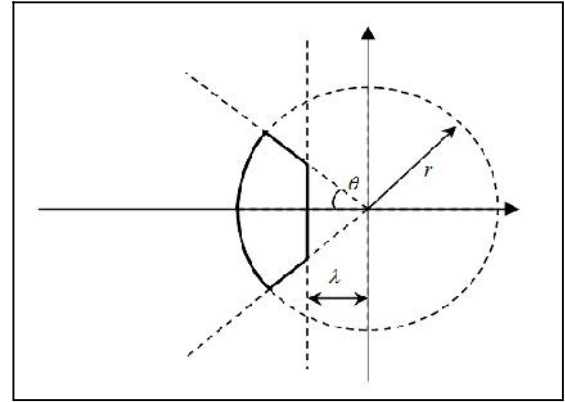


Figure 3. Region $S(\lambda, r, \theta)$.

The advantages of placing the closed loop poles to this region are the DC motor response ensures a minimum decay rate λ , a minimum damping ratio $\zeta = \cos \theta$, and a maximum undamped natural frequency $\omega_d = r \sin \theta$ [6]. Equation (16), (17) and (18) show the clustering region used in this study which are λ -stability region, a disk and the conic sector respectively.

$$M_{\mathcal{D}_1}(\tilde{A}_{cl}, X_{\mathcal{D}_1}) := \tilde{A}_{cl}X_{\mathcal{D}_1} + X_{\mathcal{D}_1}\tilde{A}_{cl}^T + 2\lambda X_{\mathcal{D}_1} < 0 \quad (16)$$

$$M_{\mathcal{D}_2}(\tilde{A}_{cl}, X_{\mathcal{D}_2}) := \begin{pmatrix} -rX_{\mathcal{D}_2} & \tilde{A}_{cl}X_{\mathcal{D}_2} \\ X_{\mathcal{D}_2}\tilde{A}_{cl}^T & -rX_{\mathcal{D}_2} \end{pmatrix} < 0 \quad (17)$$

$$M_{\mathcal{D}_3}(\tilde{A}_{cl}, X_{\mathcal{D}_3}) := \begin{pmatrix} \sin \theta (\tilde{A}_{cl}X_{\mathcal{D}_3} + \tilde{A}_{cl}X_{\mathcal{D}_3}^T) & \cos \theta (\tilde{A}_{cl}X_{\mathcal{D}_3} - \tilde{A}_{cl}X_{\mathcal{D}_3}^T) \\ \cos \theta (X_{\mathcal{D}_3}\tilde{A}_{cl}^T - \tilde{A}_{cl}X_{\mathcal{D}_3}) & \sin \theta (\tilde{A}_{cl}X_{\mathcal{D}_3} + \tilde{A}_{cl}X_{\mathcal{D}_3}^T) \end{pmatrix} < 0 \quad (18)$$

where this region is the intersection of three elementary LMI regions ($M_{\mathcal{D}1 \cap \mathcal{D}2 \cap \mathcal{D}3}(\tilde{A}_{cl}, X_{\mathcal{D}})$).

Meanwhile, the H_{∞} constraint is equivalent to the existence of a solution $X_{\infty} > 0$ to the LMI

$$\begin{pmatrix} \tilde{A}_{cl}X_{\infty} + X_{\infty}\tilde{A}_{cl}^T & X_{\infty}\tilde{C}_1^T & \tilde{B}_1 \\ \tilde{C}_1X_{\infty} & -\mathcal{I} & \tilde{D}_{11} \\ \tilde{B}_1^T & \tilde{D}_{11}^T & -\mathcal{I} \end{pmatrix} < 0 \quad (19)$$

Equation (19) is also known as the Bounded Real Lemma [12]. As mentioned before, the main objective of this study is to minimize the H_{∞} norm of $G_{yw}(s)$ over all state feedback gains K that enforce the pole constraints. However, this problem is not jointly convex in the variables $X_{\mathcal{D}1}$, $X_{\mathcal{D}2}$, $X_{\mathcal{D}3}$, X_{∞} and K . The convexity can be enforced by seeking a common solution

$$X = X_{\mathcal{D}1} = X_{\mathcal{D}2} = X_{\mathcal{D}3} = X_{\infty} > 0 \quad (20)$$

to (16)-(19) and rewriting these equations using the auxiliary variable $Y = KX$. These changes of variables lead to the suboptimal LMI approach to H-infinity synthesis with pole assignment in LMI regions. As a result, the new representations of (16)-(19) are shown in the following equation.

$$\text{Herm}[\tilde{A}X + \tilde{B}_2Y] + 2\lambda X < 0 \quad (21)$$

$$\begin{pmatrix} -rX & \tilde{A}X + \tilde{B}_2Y \\ * & -rX \end{pmatrix} < 0 \quad (22)$$

$$\begin{pmatrix} \sin\theta(\text{Herm}[\tilde{A}X + \tilde{B}_2Y]) & \cos\theta(\text{Herm}[\tilde{A}X - \tilde{B}_2Y]) \\ * & \sin\theta(\text{Herm}[\tilde{A}X + \tilde{B}_2Y]) \end{pmatrix} < 0 \quad (23)$$

$$\begin{pmatrix} \text{Herm}[\tilde{A}X + \tilde{B}_2Y] & X\tilde{C}_1^T & \tilde{B}_1 \\ * & -\mathcal{I} & \tilde{D}_{11} \\ * & * & -\mathcal{I} \end{pmatrix} < 0 \quad (24)$$

where $\text{Herm}[\tilde{A}X + \tilde{B}_2Y] = \tilde{A}X + \tilde{B}_2Y + X\tilde{A}^T + Y\tilde{B}_2^T$ and $*$ is an ellipsis for terms induced by symmetry [10]. In this study, the entire LMI problem is solved using well known LMI optimization software which is *LMI Control Toolbox*.

IV. IMPLEMENTATION RESULTS

The main objective of the feedback controller in this study is to maintain the speed of the motor with limited control

action of duty cycle. All the feedback control strategies are incorporated in the closed-loop system in order to produce the control action between 0 and 1. Regulating a DC motor from initial rest, i.e. when the initial angular velocity is zero, to some desired final angular velocity may cause very inconvenient transient behavior. In order to verify that the proposed approach allows achieving very smooth transitions, a smooth start from $\omega = 0$ to a final angular velocity (steady state) shall be planned [1]. The corresponding reference trajectory $\omega_r(t)$ for the output is associated to the boundary conditions

$$\omega_r(0) = 0 \text{ and } \omega_r(t_d) = \frac{\omega_{\max}}{2}, \quad (25)$$

from the fact that the system is transferred from one stationary point to another. The smooth trajectory which satisfies the conditions (26), is modeled as

$$\omega_r(t) = \frac{\omega_{\max}}{2} [\tanh \eta(t - t_d) + 1] \quad (26)$$

where ω_{\max} is the steady state value, t_d is shifting time and η is the constant that determine the smooth profile [13]. The smooth trajectory model has been determined such that $\omega_{\max} = 150$ rad/s, $\eta = 0.03$ and $t_d = 100$ ms.

Applying the LMI conditions in (21)-(24) with the advantage of graphical profiles, the parameter of conic sectors and disk that fulfil the design requirement is at $r = 3000$, $\lambda = -500$ and $\theta = 80^\circ$. Then, the state feedback gain, K is obtained as followed:

$$K = [-0.3732 \quad -0.5538 \quad -9.2986 \quad -2.1568 \quad 1395.95]$$

with $\gamma = 2.0192$. This state feedback gain also guarantees the H_{∞} performance $\|G_{yw}\|_{\infty} < \gamma$. The result shows that the location of poles has been confined in the selected LMI region with the value of -2805.928 , $-1693.055 \pm j2286.753$ and $-683.1136 \pm j1121.316$. As a comparative assessment, the proposed control scheme is compared with the classical PI control reported in the previous literature [14]. The simulation response of angular velocity, duty cycle input energy and armature current are depicted in Figures 4-6 respectively. It demonstrates that the proposed control schemes are capable in tracking the desired velocity from 0 to 150 rad/s. Performance of the controllers in terms of time response specifications are summarized in Table 1. It is observed that the integral square error (ISE) of the proposed controller is extremely lower as compared to the PI controller. With lower ISE, the actuator response required less time to settle down to desired velocity. It is noted that, the duty cycle for the proposed controller produced less input energy as compared to PI controller. The armature current response is illustrated in Figure 6. It shows that, both controllers produce high amplitude of armature current before settled down at 230 ms. It is noted that the proposed controller produce the maximum magnitude of

armature current with the value of 0.3487 A, as compared to PI controller with the value of 0.3418 A. On top of that, for both controllers, the buck converter supplies the highest amount of power to the motor during the transition of the angular velocity from zero to final state, since the armature currents reach their peak within the transition period.

TABLE I. COMPARATIVE ASSESSMENT OF TIME RESPONSE SPECIFICATIONS

Controller	Rise time (ms)	Settling time (ms)	ISE
Proposed	131	167	0.055
PI	127	172	0.660

V. CONCLUSION

This study has presented H-infinity with poles clustering in LMI region for speed control buck-converter driven DC motor. This LMI approach has been implemented in the LMI Control Toolbox and validated using state-space model of buck-converter driven DC motor. Performances of the controller are examined in terms of angular velocity, duty cycle input energy and armature current. The results demonstrated that the proposed controller provide high speed tracking of angular velocity with minimal usage of duty cycle input energy.

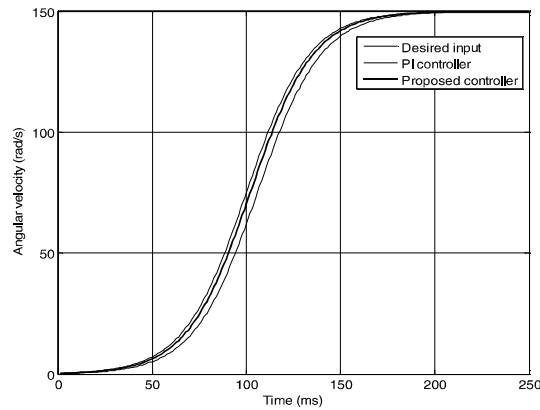


Figure 4. Angular velocity response.

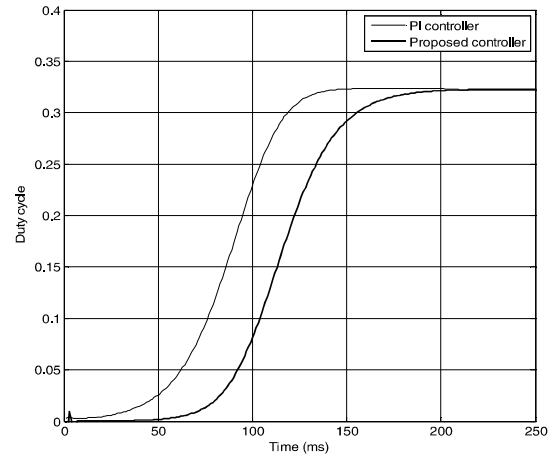


Figure 5. Duty cycle response.

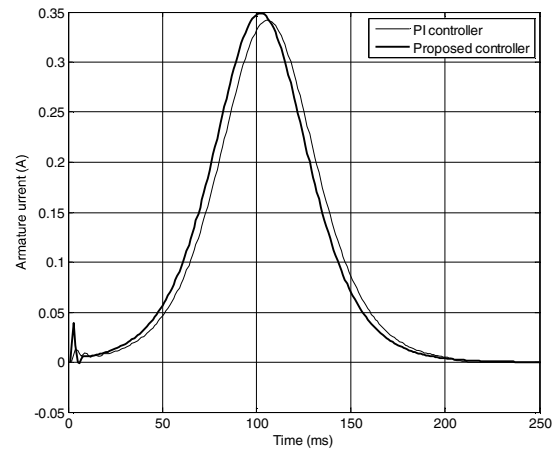


Figure 6. Armature current response.

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